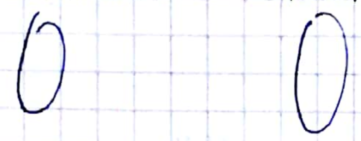
22-Apr-2018 **Pigeon-hole principle.**

Assume a function

f : Domain → Codomain



and f is onto., and $|\text{codomain}| \le |\text{domain}|$

Least number (we are 100% sure) where m elements in the domain have the same image in the co-domain.

$$m = \left\lceil \frac{|\text{domain}|}{|\text{codomain}|} \right\rceil$$

Question: We have 37 students in MTH 213. Convince me:

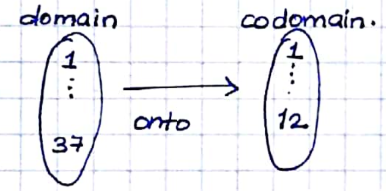
i) At least 4 students are born on the same month.

ii) At least 6 students are born on the same day. of the week.

Assume all students are born betw. ~~2000 – 2018~~ ~~2017.~~ 1990 – 1999 Convince me

iii) At least ~~3~~ 4 students are born on the same year.

Answers: i) $|\text{domain}| = 37$ $|\text{co domain}| = 12$



* Note: Assume, think about fair share

least # of students born in same day $= \left\lceil \frac{37}{12} \right\rceil = 4$

ii) $|\text{codomain}| = 7$ $|\text{domain}| = 37$

least no. of students born on same day $= \left\lceil \frac{37}{7} \right\rceil = 6.$

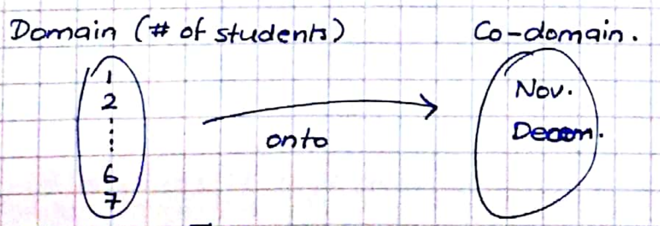
iii) $|\text{codomain}| = 10$ $|\text{domain}| = 37.$

least no. of students born on same year $= \left\lceil \frac{37}{10} \right\rceil = 4$

24-Apr-2018. Question: Assume 7 students, born on either November or December. At least m out of 7 students born on same month. Find max. value of m.

Find min value of m = 0

Ans:



$$m = \left\lceil \frac{|\text{domain}|}{|\text{codomain}|} \right\rceil = \left\lceil \frac{7}{2} \right\rceil = 4.$$

Means, on ~~eit~~ one of the given months, at least 4 students were born.

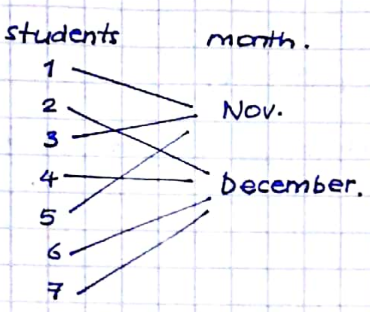
ii) There is a month where at most $\underline{n}$ students were born. Find min. value of n.

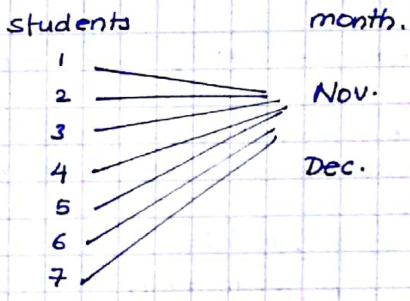
Find max value of $n = 7$

Ans) $n = \left\lfloor \frac{7}{2} \right\rfloor = 3.$

Means, in a given month, (with all possibilities tried), no ~~less~~ more than 3 ppl were born in that month.

eg. functions:





$m = 4$: At least 4 born in december
$n = 3$: At most 3 born in november

$m = 4$: At least 4 born in november
$n = 3$: At most 3 born in december

24-Apr-2018 **Permutation and Combination.**

Permutation : order is important.
Combination : order is not important.

$$\binom{n}{r} = nCr = \frac{n!}{(n-r)!\,r!}$$

"n choose r"
From n objects, choose r objects out of n randomly (order not important).

eg $\binom{5}{3} = 5C3 = \frac{5!}{2!\,3!}$

Note: Pascal's triangle:

n=1.	1
n=2	1 2 1
n=3	1 3 3 1
n=4	1 4 6 4 1
	↑ ↑ ↑ ↑ ↑
	4C0 4C1 4C2 4C3 4C4

Fact: $\binom{n}{r} = \binom{n}{n-r}$

$\therefore$ 10C3 = 10C7.

Suppose $(a+b)^n$, $n \in N^*$
then $(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots \binom{n}{n-1} a^1 b^{n-1} + b^n$

$= \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots \binom{n}{n-1} a^1 b^{n-1} + \binom{n}{n} a^0 b^n.$

prove: $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots \binom{n}{n} = 2^n.$

26-Apr-2018:

Eg: $20C3 = \frac{20!}{3!\,17!} = \frac{20 \times 19 \times 18}{3 \times 2 \times 1}.$

Question: Suppose $(a+b)^{30}$. Give me the coefficient of $a^2 b^{28}$.
Ans: coef = 30C28 = 30C2

Question: Suppose there are 22 students in MTH 213. Ayman shakes hand w/ everyone. Every student shakes hand w/ every other student exactly once. How many hand shakes are there?
Ans: 22C2 + 22?

Question: A bike lock has 1 digit combination.
1st digit = 3 − 7
2nd digit = 1 − 8
3rd digit = 5 − 9
4th digit = 2 − 7. Find total number of all possible combinations.
Ans: 5 × 8 × 5 × 6 = 1200 combinations.

Question: Same as above, each digit is a number 1-9 and no repetition.

Ans: $9P4 = 9 \times 8 \times 7 \times 6 = \frac{9!}{(9-4)!} = 3024$

Question: Imagine there are 6 holes and 4 balls of distinct colors (red, yellow, green, white). Put each of the 4 balls in one of the holes. ~~We~~

Ans: We randomly choose 4 holes (combination). Then apply the permutation

$\underbrace{(6C4)}_{\text{choose 4 holes out of 6.}} \cdot (4P4)$

and

Question: Assume there are 22 students, we need to form a commitee of 4 people.

Ans: 22 C 4 (choose 4 out of 22 randomly).

Question: Assume 22 students, 10 female, 12 Male. There are 4 available positions

President	Prov	Asst. Prov	Secretary
(has to be male)	(male)	(female)	(~~fea~~ female).

How many possible outcomes?

Ans: $\underbrace{(12P2)}_{\text{outcomes among males}} \times \underbrace{(10P2)}_{\text{outcomes among females}}$

→ order is important. $(A, Y) \neq (Y, A)$